

Conceptual and Application of Properties of Integer Operations in Elementary School Mathematics Learning

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Article Info :	ABSTRACT
Accepted: 20-12-2025	Background: Integers constitute one of the fundamental concepts in mathematics and play an important role in everyday life as well as in elementary school mathematics learning. A solid understanding of integers is essential because it forms the basis for learning more advanced mathematical concepts. However, many students still encounter difficulties in understanding the concept of integers and performing integer operations. Objective: This study aims to analyze the concept of integers, the properties of integer operations, and their implementation in mathematics learning. Method: The research employed a literature review method by examining various books and scientific journals related to mathematics education. Result: The results of the analysis indicate that students' understanding of integers can be improved through contextual learning approaches, the use of concrete learning media, and a systematic explanation of the properties of integer operations. These findings highlight the importance of designing meaningful learning experiences that connect mathematical concepts with students' real-life situations. Implication: This study concludes that a strong conceptual understanding of integers is crucial for elementary school students and can be effectively supported through contextual teaching strategies and the use of appropriate instructional media.
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INTRODUCTION

Mathematics is one of the fundamental subjects taught at the elementary school level because it plays an important role in developing students' logical thinking, problem-solving abilities, and analytical skills. One of the basic topics that must be mastered by students is the concept of integers. Integers are used in various real-life situations, such as measuring temperature, calculating financial gains and losses, and determining positions above or below sea level.

Therefore, a clear understanding of integers is essential as a foundation for learning more advanced mathematical concepts.

However, in practice, many elementary school students still encounter difficulties in understanding integer concepts and performing integer operations. These difficulties often arise because students tend to memorize procedures without fully understanding the underlying concepts. In addition, the learning process sometimes relies heavily on abstract explanations without the support of concrete learning media or contextual examples. As a result, students may struggle to visualize the meaning of positive and negative numbers as well as the relationships between them. This situation indicates the importance of improving learning approaches so that students can develop a deeper conceptual understanding of integers.

Several previous studies have discussed the learning of integers in elementary mathematics education. In addition, a study conducted by Jo Boaler emphasizes that meaningful mathematical learning should involve contextual experiences and conceptual understanding rather than rote memorization. These studies highlight the importance of designing learning strategies that help students construct their own understanding of mathematical concepts, including integers.

Research conducted by SQ Ain, (2021) reveals that mathematics learning difficulties at the elementary school level are predominantly influenced by both internal and external factors, including students' low motivation, limited understanding of abstract concepts, and insufficient use of concrete learning media. Similarly, Bengue et al., (2021) identified that a significant number of students encounter systematic difficulties when solving integer operation problems, particularly those involving operations with negative numbers, indicating that integer concepts require more structured and targeted instructional approaches from the very beginning of mathematics education at the elementary level.

Empirical evidence further highlights the persistent nature of students' difficulties in mastering integer operations. Nurnberger-Haag et al., (2022) developed and validated an integer test specifically designed to assess students' computational competence with negative numbers, revealing widespread and consistent gaps across educational levels. Complementarily, Harun et al., (2023) assessed students' mastery and common misconceptions in the fundamental operations on integers and identified recurring errors in subtraction and multiplication involving negative values, suggesting that these concepts require more deliberate conceptual instruction rather than procedural drill alone. From a theoretical standpoint, the distinction between

conceptual knowledge and procedural knowledge is particularly significant in mathematics learning.

Braithwaite & Sprague, (2021) demonstrated that conceptual knowledge plays a foundational role in supporting both routine and nonroutine problem solving, with metacognitive processes mediating the relationship between the two types of knowledge. This is supported by Kholid et al., (2021), who found that students' conceptual understanding is a critical predictor of mathematical problem-solving performance. In the context of elementary mathematics, Nahdi et al., (2020) demonstrated that students frequently develop procedural fluency before establishing deep conceptual understanding, often resulting in fragile knowledge structures that hinder flexible and meaningful application of mathematical concepts.

Learning obstacles in elementary mathematics are not limited to integer topics but reflect broader patterns across the curriculum. Hariyani et al., (2022) categorized learning obstacles faced by elementary students in mathematical topics into ontogenetic, didactical, and epistemological types, suggesting that instructional approaches must be adapted to address each type systematically. Contextual and realistic approaches have demonstrated particular promise in this regard: Sari et al., (2022) showed that a Realistic Mathematics Education approach significantly improved students' understanding of mathematical operations in elementary school by connecting abstract concepts to meaningful real-life contexts.

In parallel, Sutarni et al., (2023) found that the integration of Higher-Order Thinking Skills (HOTS) with realistic learning models effectively strengthens students' mathematical problem-solving abilities, indicating that conceptual depth and contextual relevance are complementary and mutually reinforcing dimensions of quality mathematics instruction. Although previous research has discussed students' difficulties and the importance of conceptual learning, studies that specifically analyze the concept of integers, the properties of integer operations, and their application in elementary mathematics learning through a literature review approach are still limited.

Therefore, the novelty of this study lies in its effort to systematically examine the theoretical foundations of integer concepts and the properties of integer operations, as well as to identify effective learning approaches that can support students' understanding in elementary school mathematics learning. Based on this background, the purpose of this study is to analyze the concept of integers, examine the properties of integer operations, and explore their implementation in mathematics learning through a review of relevant literature. The findings of this study are expected to provide theoretical

insights and practical references for teachers in designing more effective mathematics learning activities related to integer concepts.

RESEARCH METHOD

This study employed a field research method to obtain data directly from the learning environment in an elementary school. The research was conducted at an elementary school in Pegajahan, North Sumatra, Indonesia. Field research was chosen to provide an authentic description of how the concept of integers and the properties of integer operations are taught and understood by students in real classroom settings.

This study used a qualitative research approach aimed at gaining an in-depth understanding of students' conceptual comprehension and the implementation of integer operation properties during the mathematics learning process. The qualitative approach allows researchers to explore students' thinking processes, learning difficulties, and classroom learning dynamics in a more comprehensive manner.

The selection of a qualitative research approach in this study is grounded in well-established principles of educational inquiry. In the Indonesian educational research context, The research subjects were fourth-grade elementary school students, consisting of 20 students who participated in mathematics learning related to integer concepts. The participants were selected because integer concepts are first introduced systematically at this grade level. The students had diverse levels of mathematical ability, which allowed the researcher to observe variations in students' conceptual understanding and problem-solving strategies.

Data were collected through classroom observations and students' responses to conceptual questions related to integer operations. The data were obtained in the form of descriptive field notes, students' written answers, and explanations provided during the learning activities. These data were used to identify patterns of students' understanding and the instructional practices used by the teacher.

The data analysis technique followed the qualitative analysis procedures proposed by Matthew B. Miles and A. Michael Huberman, which consist of three main stages: data reduction, data presentation, and conclusion drawing. In the data reduction stage, the collected data were selected and organized to focus on information relevant to students' understanding of integer concepts. In the data presentation stage, the data were described systematically in narrative form to illustrate patterns of understanding and learning difficulties. Finally, conclusions were drawn by interpreting the findings to explain how conceptual and operational aspects of integers are developed in elementary mathematics learning.

Research Instrument (Conceptual Questions)

To examine students' understanding of integer concepts and operations, the researcher used several contextual questions designed to encourage higher-order thinking skills (HOTS).

Question 1: Addition of Positive and Negative Numbers

In a game, Beni earns 6 points. Then he loses 2 points.
How many points does Beni have now?

HOTS Indicator:

Students analyze a real-life situation and represent it using integer addition and subtraction. They interpret the meaning of gain and loss in mathematical form.

Question 2: Additive Inverse Concept

Ani has 4 candies. She gives 4 candies to her younger sibling.

- How many candies does Ani have now?
- Why does the result become zero?

HOTS Indicator:

Students not only determine the result but also explain conceptually why a number and its opposite produce zero, demonstrating understanding of the additive inverse concept.

Question 3: Application of Negative Numbers in Daily Context

Rudi has Rp5,000. He wants to buy a pencil that costs Rp7,000.

- Is Rudi's money enough?
- If not, how much money is he lacking?

HOTS Indicator:

Students apply integer concepts to a financial context and interpret shortage as a negative value. They reason quantitatively about deficit situations.

Question 4: Multiplication of Integers in Context

An ant moves down 2 steps every minute.

If the ant moves down for 4 minutes, how many steps has it moved down from the starting point?

HOTS Indicator:

Students model repeated movement as multiplication and interpret downward movement as a negative direction, connecting conceptual understanding with procedural skills.

Question 5: Comparing Integers

On the scoreboard, the following scores are written:

- Dita: -1

- b. Siska: 3
- c. Whose score is greater?
- d. Explain your answer.

HOTS Indicator:

Students compare integers and justify their reasoning using number line concepts or relational understanding of positive and negative numbers.

RESULT AND DISCUSSION

Research Findings

Based on classroom observations conducted at the elementary school in Pegajahan, North Sumatra, it was found that the teacher employed a combination of direct instruction and contextual examples to introduce integer concepts. The teacher primarily used a number line drawn on the whiteboard as a visual aid when explaining the positions of positive and negative integers. Contextual situations, such as temperature readings and financial gain-loss scenarios, were frequently used to illustrate abstract integer concepts. However, the instructional approach was predominantly teacher-centered, with limited opportunities for students to independently explore or construct their own understanding of integer concepts.

The analysis of students' written responses to the five Higher-Order Thinking Skills (HOTS) conceptual questions yielded the following results. On Question 1 (addition involving positive and negative integers in a game context), 16 out of 20 students (80%) answered correctly, demonstrating an ability to apply integer addition in a familiar situational context. On Question 2 (the concept of additive inverse), only 10 students (50%) could both calculate the correct result and provide a conceptual explanation of why the sum of a number and its opposite equals zero, indicating a significant gap between procedural and conceptual understanding.

On Question 3 (application of negative numbers in a financial context), 14 students (70%) correctly identified the deficit and expressed it as a negative value. On Question 4 (multiplication of integers in a movement context), 12 students (60%) correctly modeled repeated downward movement as integer multiplication, though several students struggled to interpret the directionality as a negative value. On Question 5 (comparison of integers using the number line), 15 students (75%) correctly identified the larger integer and provided a brief justification based on position on the number line.

Discussion

The findings reveal that students performed relatively well on contextual problems involving basic integer operations (Questions 1 and 3), yet showed noticeably weaker understanding on tasks requiring deeper conceptual reasoning, particularly regarding additive inverses (Question 2) and the interpretation of multiplication in a directional context (Question 4). The 50% success rate on Question 2 strongly suggests that procedural fluency does not

automatically transfer to conceptual understanding of the additive inverse property, a finding that underscores the importance of explicit conceptual instruction. Furthermore, the observation that students performed markedly better on contextually framed problems (Questions 1 and 3) than on more abstract operational tasks aligns with Boaler, (2022) assertion that meaningful mathematical learning should be grounded in contextual experiences rather than rote memorization.

This suggests that contextualizing integer problems within real-life scenarios—such as games, financial transactions, and physical movement—can significantly enhance student engagement and comprehension. The classroom observation further corroborated this finding, as students appeared more engaged and responsive when the teacher introduced problems embedded in everyday contexts. The variation in performance across the five question types also indicates that higher-order thinking skills in mathematics are not uniformly distributed among fourth-grade students. This is consistent with the findings of (Retnawati et al., 2018), who emphasized that HOTS in mathematics require deliberate instructional scaffolding. The use of learning media, such as number lines and concrete manipulatives, could further support students in constructing meaningful understanding of integer operations. The following section provides the theoretical framework underlying the properties of integer operations as observed and discussed in this study.

The observed gap between procedural and conceptual understanding, particularly evidenced by the 50% success rate on Question 2 regarding additive inverses, is consistent with theoretical accounts of knowledge acquisition in mathematics. Barumbun & D Kharisma, (2022) argue that mathematics learning should ultimately aim for proceptual knowledge—an integrated form of understanding that combines procedural efficiency with deep conceptual insight. When students possess only procedural fluency without the corresponding conceptual foundation, their knowledge remains fragile and context-dependent. This is reinforced by Nahdi et al., (2020), who demonstrated that elementary school students who develop procedural fluency prior to establishing conceptual understanding are more prone to making systematic errors when problems are presented in novel or abstract forms, as was observed in students' responses to the additive inverse and directional multiplication questions in this study.

The role of the number line as a concrete and visual model in supporting integer learning merits particular attention. Bouck et al., (2021) demonstrated that the use of a virtual number line with corrective feedback significantly improved middle school students' ability to perform integer addition operations, highlighting the cognitive affordances of spatial and sequential visual representations in constructing integer understanding. In the present study, the teacher's use of a number line drawn on the whiteboard served a similar scaffolding function, providing students with a tangible referent for visualizing integer relationships and positions. However, Hasanah & H Retnawati, (2022) caution that the effectiveness of contextual learning in

mathematics depends not only on the availability of visual aids but equally on the quality of pedagogical implementation and the coherence between learning activities and students' cognitive developmental readiness.

The relationship between linguistic representation and mathematical understanding also bears relevance to the present findings. Lin et al., (2021) conducted a comprehensive meta-analysis demonstrating that students' mathematical vocabulary knowledge is significantly and positively associated with mathematics performance, with effect sizes particularly pronounced in problem-solving tasks requiring conceptual understanding and explanation. This is reflected in the present study's findings: students who were able to articulate conceptual reasoning in their responses—such as explaining why a number and its opposite sum to zero in Question 2 or justifying integer comparison in Question 5—tended to demonstrate stronger overall performance. These results suggest that integrating explicit attention to mathematical language and conceptual vocabulary into integer instruction at the elementary level can meaningfully support deeper understanding.

The students' varied HOTS performance across the five question types in this study is further contextualized by prior research on higher-order thinking in elementary mathematics. Saraswati & GNS Agustika, (2020) found that elementary school students' ability to solve HOTS mathematics questions was generally at a moderate level, with analysis and evaluation skills showing greater challenges than knowledge and comprehension levels as categorized by Bloom's revised taxonomy. More recently, Sulistyawati et al., (2023) demonstrated that integrating HOTS-oriented problems into problem-based learning significantly enhanced students' mathematical problem-solving abilities in ways that procedural-only exercises could not achieve, and Kholid et al., (2021) confirmed that conceptual understanding is a critical antecedent for students' capacity to engage successfully with higher-order mathematical tasks. Collectively, these findings reinforce the recommendation that integer instruction in elementary schools should deliberately incorporate HOTS-level tasks that encourage students to move beyond procedural recall toward genuine conceptual reasoning, mathematical justification, and flexible problem-solving.

Theoretical Review of Integer Concepts and Properties of Operations

Integers are a set of numbers consisting of whole numbers and their negative counterparts. Whole numbers include 0, 1, 2, 3, and so on, while negative numbers are their opposites, such as -1, -2, -3, and so forth. The number -0 is not treated separately because it has the same value as 0. In general, integers are classified into three categories: positive integers, zero, and negative integers. This classification helps students understand the structure of integers systematically in mathematics learning. Every integer has exactly one additive inverse. Two numbers are considered opposites if their sum equals zero, which can be expressed as $a + (-a) = 0$ or $0a + (-a) = 0$ and $0a + (-a) = 0$. For example, the opposite of 4 is -4, the opposite of -7 is 7, and the opposite of

0 is 0. Understanding additive inverses is fundamental in learning integer addition and subtraction.

In real-life contexts, negative numbers represent values below zero or deficit conditions, such as temperatures below 0°C , altitudes below sea level, or financial debt. Positive numbers, on the other hand, represent quantities above zero, such as profit or the number of objects. On a number line, positive numbers are located to the right of zero, while negative numbers are located to the left. The number line serves as a visual representation to clarify the order of integers. Integers satisfy properties of equality relations, including reflexive, symmetric, and transitive properties. They also follow order relations, where numbers greater than zero are positive and numbers less than zero are negative. Understanding these relations supports students in comparing and analyzing integers logically.

Integer operations include addition, subtraction, multiplication, and division. Addition satisfies the commutative, associative, identity, inverse, and closure properties, whereas subtraction does not satisfy commutative and associative properties. In multiplication, integers with the same sign produce a positive result, while integers with different signs produce a negative result. Division is the inverse of multiplication but is undefined when the divisor is zero. Conceptually, integer operations can be introduced using visual models such as number lines and concrete media like colored cards to help students develop meaningful understanding.

Table 1. Properties of Integer Operations

Operation	Commutative	Associative	Identity Element	Inverse Exists
Addition	Yes	Yes	0	Yes
Subtraction	No	No	None	No
Multiplication	Yes	Yes	1	Yes (except 0)
Division	No	No	None	No (if divisor = 0)

Source: Data Processed



Figure 1. Integers

CONCLUSION

Based on the results of field research and data analysis, elementary school students demonstrate varying levels of conceptual understanding of integers and their operations. Most students perform better when integer concepts are presented through contextual situations such as games, financial transactions, or movement directions, while some still struggle with interpreting negative numbers and understanding opposite numbers. The study concludes that strengthening students' conceptual understanding requires learning strategies that integrate contextual problems, conceptual explanations of integer operations — including additive inverse, comparison of integers, and interpretation of negative values — and opportunities for students to express their reasoning, so that mathematics learning at the elementary school level prioritizes meaningful conceptual understanding over procedural calculations alone.

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