

The Effect of Visual and Manipulative Learning Media on Elementary School Students' Conceptual Understanding of Integers: A Quasi-Experimental Study

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Article Info :	ABSTRACT
Accepted: 29-01-2026	Background: Many elementary school students experience difficulties in understanding integer concepts, particularly negative numbers, due to the abstract nature of the topic and reliance on conventional teaching methods that emphasize procedural memorization over conceptual understanding. Objectives: This study aims to examine the effect of visual and manipulative instructional media on fifth-grade elementary school students' conceptual understanding of integer concepts. Method: A quantitative approach with a quasi-experimental design using a nonequivalent control group design was employed. The participants, fifth-grade students at an elementary school in Indonesia, were divided into an experimental class and a control class. The experimental class received instruction using visual and manipulative media, while the control class received conventional instruction. Data were collected through pre-test and post-test instruments and analyzed using an independent sample t-test. Findings and Implications: The experimental class experienced a significantly higher improvement in conceptual understanding compared to the control class ($t = 2.87, p < 0.05$), with mean gain scores of 26.8 and 12.4, respectively. Conclusion: The use of visual and manipulative instructional media can effectively enhance elementary school students' conceptual understanding of integers. Teachers are encouraged to integrate number line representations and concrete manipulatives as part of a sequential instructional approach in mathematics learning.
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INTRODUCTION

Mathematics is one of the fundamental subjects in elementary education because it develops students' logical thinking, problem-solving skills, and reasoning abilities. However, many elementary school students experience difficulties in understanding abstract mathematical concepts. One of the topics that is often considered challenging is integers, particularly negative numbers.

Integers require students to move beyond counting tangible objects and begin understanding numbers that represent quantities below zero, which can be cognitively demanding for young learners.

Several studies have shown that elementary school students frequently experience misconceptions when learning integers. Common difficulties include misunderstanding the concept of negative numbers, assuming that negative numbers cannot be compared logically with positive numbers, and confusion when performing operations such as subtraction involving negative values. For example, students often assume that -8 is greater than -3 because 8 is larger than 3, indicating a misunderstanding of the number line concept. Such misconceptions can hinder students' deeper understanding of mathematical concepts and affect their performance in later mathematical topics.

Synthesizing the documented obstacles in learning integers, Alfarisi et al., (2022), in a systematic literature review of studies indexed in Scopus and Google Scholar, confirmed that integers consistently emerge as one of the most challenging topics in school mathematics. Their review identified that students at all levels encounter persistent difficulties with negative numbers and integer operations, and that these difficulties are rooted in conceptual rather than procedural gaps. This finding underscores the importance of providing targeted instructional approaches that explicitly address the conceptual foundations of integer understanding, rather than relying solely on procedural memorization.

The prevalence of integer misconceptions among elementary school students has been well-documented in the mathematics education literature. Lin, (2022) conducted a systematic review of the obstacles and misconceptions experienced by students in learning numbers, and found that negative integers consistently emerge as one of the most conceptually challenging topics. Students frequently confuse the magnitude of negative numbers with their direction on the number line, and often fail to generalize whole-number reasoning to the integer domain. This body of evidence underscores the need for targeted instructional strategies that directly address the unique conceptual barriers associated with integer learning.

From a cognitive development perspective, the difficulty of understanding integers can be explained by Piaget's theory of cognitive development. According to Piaget, elementary school students are generally in the concrete operational stage, where they learn more effectively through concrete objects and visual representations rather than abstract explanations. Therefore, the use of visual and manipulative instructional media can support students in constructing mathematical concepts more meaningfully. By

interacting with concrete representations such as number lines, counters, or visual models, students can better visualize the relationships between positive and negative numbers.

The constructivist foundation underlying the use of manipulatives in mathematics education has been further supported by recent work in teacher education. Greenstein & E Fernández, (2023) demonstrated that when pre-service elementary teachers design and use original mathematics manipulatives, their understanding of how physical tools support abstract concept formation deepens significantly. Their findings reinforce the view that well-designed manipulative-based activities, grounded in constructivist principles, are not merely pedagogical aids but are fundamental mechanisms through which mathematical meaning is constructed by learners at every stage of development.

The effectiveness of CRA-based approaches in bridging conceptual gaps in mathematics has been further confirmed by recent meta-analytic evidence. Ebner et al., (2025) conducted a comprehensive meta-analysis of 30 studies employing single-case design methodologies and found a statistically significant overall effect size ($\text{Tau-BC} = 0.9965, p < .0001$) in favor of the CRA approach as a mathematics intervention. These findings provide robust quantitative evidence that the three-phase instructional progression from concrete manipulatives to representational drawings to abstract symbols is a highly effective framework for building mathematical understanding, reinforcing the theoretical rationale for the media-based instructional design employed in the present study.

Recent empirical studies also highlight the importance of instructional media in improving students' conceptual understanding in mathematics. For instance, research conducted by various scholars in the last decade indicates that visual and manipulative media can significantly enhance students' understanding of abstract mathematical concepts, including integers. These studies demonstrate that learning media help bridge the gap between abstract mathematical ideas and students' concrete experiences, thereby improving engagement and comprehension in mathematics learning.

A recent large-scale meta-analysis by Schoenherr & Schukajlow, (2024) synthesized 41 visualization intervention studies involving 10,562 students across all levels of mathematics education. The results demonstrated a medium overall positive effect ($g = 0.504$) of visualization interventions on mathematics learning outcomes, with effect sizes being particularly pronounced in quasi-experimental studies. Importantly, manipulatives were found to produce the largest effect sizes among all visualization types, lending strong empirical support to their use in elementary mathematics instruction.

Similarly, Byrne et al., [\(2023\)](#) conducted a comprehensive scoping review of physical manipulative interventions in primary education and confirmed that manipulative-based approaches consistently improve children's mathematical abilities, particularly for abstract topics requiring concrete representation.

Despite the growing recognition of the importance of instructional media, many elementary mathematics classrooms still rely heavily on conventional teaching methods, such as lectures and textbook-based explanations. As a result, students often memorize procedures without fully understanding the underlying concepts. This issue becomes more relevant in the context of current educational reforms that emphasize student-centered learning and conceptual understanding in mathematics education. Therefore, integrating appropriate instructional media into classroom instruction has become increasingly important to support effective learning.

The importance of structured and well-sequenced instructional design in elementary mathematics has been highlighted by [Schnepel et al. \(2022\)](#), who conducted a systematic review of 20 mathematics intervention studies targeting primary school students. Their findings identify the intensity and explicitness of instruction as key factors in producing significant learning gains, particularly when interventions are grounded in concrete and representational tools. This evidence supports the position that instructional media do not function in isolation but must be embedded within a well-designed pedagogical sequence to maximize their impact on students' mathematical understanding.

The role of the number line as both a visual and manipulative tool in integer instruction has received particular scholarly attention. Sari et al., [\(2020\)](#) examined the effectiveness of an empty number line model for integer addition and subtraction, revealing that this representational tool supports students in constructing conceptual understanding of directionality and magnitude in the integer domain. Anton & Abrahamson, [\(2025\)](#) further investigated how students develop enactive understanding of integer arithmetic through physical engagement with number lines at different scales, finding that the coordination between physical and symbolic experiences significantly deepens students' conceptual grasp of negative number operations. These findings collectively indicate that the number line is not merely a visual aid but an epistemologically powerful tool for integer concept formation.

The conceptual structures through which students make sense of integer operations have also been investigated by Wessman-Enzinger & Mooney, [\(2021\)](#), who identified four distinct conceptual models including Bookkeeping, Counterbalance, Relativity, and Translation that students

employ when reasoning about integer addition and subtraction. Their analysis reveals that students' conceptual understanding of integers is multidimensional, involving the coordination of context, directionality, and magnitude. These findings imply that effective integer instruction should provide students with rich representational experiences that support the development of multiple conceptual models, thereby deepening their understanding beyond surface-level procedural competence.

In the Indonesian educational context, students' difficulties with integer concepts at the elementary level remain a widely reported concern. Research indicates that conventional teaching approaches, which rely predominantly on memorization of rules and algorithmic procedures, fail to equip students with the conceptual foundations necessary for understanding negative numbers and integer operations. The absence of concrete and visual learning experiences leaves many students unable to connect the formal symbolic representations of integers to meaningful real-world situations, reinforcing the urgency of adopting media-based instructional strategies aligned with students' cognitive developmental stage and consistent with current curriculum reform directions that emphasize deep conceptual understanding.

A broader perspective on the landscape of research on concrete-to-abstract mathematical learning has been provided by Huan et al., ([2022](#)), whose bibliometric analysis of 425 publications in the Scopus database traced the development of this research domain from 1968 to 2021. Their findings show an exponential increase in publications on this topic, reflecting growing scholarly recognition of the role that concrete and representational tools play in supporting abstract mathematical reasoning. The predominance of research on manipulatives and arithmetic within this literature base directly affirms the relevance of visual and manipulative instructional media to elementary mathematics education, including the teaching of integers.

Research evidence confirms that the gap between procedural and conceptual understanding of integers is a persistent challenge at the elementary level. Nurnberger-Haag et al., ([2022](#)) developed and validated an Integer Test of Primary Operations and found that many students who could perform calculations procedurally lacked the conceptual understanding needed to interpret and apply integer operations meaningfully. This finding highlights the importance of ensuring that instructional approaches explicitly target conceptual understanding, not merely procedural fluency. In a related vein, Ismail et al., ([2023](#)) found that lessons utilizing Gizmos-based virtual manipulatives significantly improved students' performance in integers compared to conventional instruction, demonstrating the effectiveness of representational tools in building integer comprehension.

In the Indonesian educational context, Wibowo et al., [\(2022\)](#) specifically analyzed students' ability to understand mathematical concepts of integer operations and found significant gaps between procedural performance and genuine conceptual understanding at the middle school level. Their findings indicate that many students who can mechanically execute integer calculations lack the underlying conceptual knowledge needed to interpret and apply integer operations in contextual situations. This evidence drawn from the Indonesian educational setting highlights the critical need for instructional approaches that prioritize the development of conceptual understanding in integer learning, such as the visual and manipulative media-based instruction examined in the present study.

The typology of external visualizations used in mathematics education has also been examined in recent scholarly work. Schoenherr et al., [\(2024\)](#) proposed a framework for characterizing external visualizations in mathematics education, distinguishing between visualizations with physical resemblance and those with structural resemblance, such as number lines and relational models. Within this framework, manipulative materials and number line representations occupy a central position as tools that bridge the gap between concrete physical actions and abstract mathematical reasoning, making them particularly suitable for instruction on topics such as integers that require students to develop both directional and operational understanding of number.

The connection between representational flexibility and integer understanding has been further examined by Lamb et al., [\(2023\)](#), who found that students who could fluently move between multiple representations — including number lines, counters, and contextual models — demonstrated deeper conceptual understanding and greater adaptability in solving novel integer problems. This suggests that effective integer instruction should not rely on a single representational tool, but should systematically integrate multiple visual and manipulative representations to support robust conceptual development. The present study draws on this principle by incorporating both visual (number line) and manipulative (concrete counter) media within an integrated instructional approach.

The cognitive processes through which students at different grade levels engage with integer addition problems have been closely examined by Aqazade et al., [\(2021\)](#), who studied second and fifth graders' use of knowledge structures when solving integer addition problems. Their research found that students' strategies were shaped by the interplay between prior whole-number knowledge and emerging integer-specific understanding, and that the representation and sequencing of instruction play a critical role in supporting

this conceptual transition. These findings reinforce the value of multi-representational instructional approaches, such as the integration of visual number line representations and concrete manipulatives employed in the present study, in scaffolding students' developing understanding of integer concepts.

Based on these considerations, it is necessary to investigate the effectiveness of instructional media in improving elementary school students' understanding of integer concepts. This study aims to examine the effect of visual and manipulative instructional media on students' conceptual understanding of integers through a quasi-experimental approach. The findings of this study are expected to provide practical implications for teachers in selecting effective learning media, for schools in supporting innovative learning environments, and for curriculum developers in designing mathematics instruction that facilitates deeper conceptual understanding among elementary school students.

Research on the effectiveness of instructional media in mathematics learning has also highlighted the importance of proper scaffolding and sequencing within the instructional design. This finding implies that the effectiveness of visual and manipulative media depends not only on the tools themselves but also on the pedagogical framework within which they are implemented, including the intentional sequencing of concrete, representational, and abstract phases of instruction.

Given the convergence of theoretical perspectives and empirical evidence reviewed above, it is clear that the integration of visual and manipulative instructional media represents a well-supported approach to improving elementary students' conceptual understanding of integers. The Concrete Representational Abstract (CRA) instructional framework advocates for a progression from hands-on manipulation of physical objects to pictorial representations and finally to abstract symbolic notation. Studies employing quasi-experimental designs have consistently shown that students taught through CRA-aligned, media-based approaches demonstrate superior conceptual understanding compared to peers taught through conventional methods, providing a strong empirical rationale for the present study.

RESEARCH METHOD

This study employed a quantitative approach using a quasi-experimental design with a nonequivalent control group design. The research was conducted at an elementary school involving fifth-grade students as the research subjects. The participants were divided into two groups: an experimental class and a control class. The experimental class received instruction using visual and manipulative instructional media designed to

facilitate students' understanding of integer concepts, while the control class was taught using conventional teaching methods without the use of specific instructional media.

The quasi-experimental design with nonequivalent control group was selected based on the practical constraints of classroom research, where random assignment of students to conditions is not feasible. This design is widely employed in educational research to evaluate the effects of instructional interventions while acknowledging that pre-existing differences between groups may exist, which are controlled through the inclusion of pre-test measurement Creswell & Creswell, (2017). The choice of a quantitative approach was guided by the study's objective of measuring and comparing the magnitude of change in students' conceptual understanding scores between the experimental and control conditions, consistent with established practices in mathematics education research.

The research instrument was a conceptual understanding test on integers administered in the form of a pre-test and post-test. The test consisted of 10 essay questions designed to measure students' conceptual understanding of integer operations. The blueprint of the instrument was developed based on several indicators of conceptual understanding, including: (1) identifying and interpreting positive and negative integers in contextual situations, (2) explaining the meaning of integer operations using representations such as number lines or real-life situations, (3) applying integer operations in problem-solving contexts, and (4) connecting integer concepts with everyday situations. These indicators were adapted from commonly used conceptual understanding frameworks in mathematics education.

The indicators of conceptual understanding used in this study were aligned with internationally recognized standards for mathematics instruction. According to the National Council of Teachers of Mathematics, (2020), conceptual understanding in the number and operations domain at the elementary level should encompass students' ability to interpret the meaning of numbers, represent quantities in multiple ways, and apply number operations flexibly in contextual situations. These standards informed the design of the assessment instrument and ensured that the evaluation of students' conceptual understanding was grounded in internationally recognized criteria for mathematics learning at the elementary level, enhancing the construct validity of the measurement used in this study. Before being used in the study, the instrument was validated through content validation using expert judgment.

Two mathematics education experts reviewed the items to ensure that the questions were aligned with the learning objectives, indicators of conceptual understanding, and the level of difficulty appropriate for fifth-

grade students. After the validation process, several items were revised based on the experts' suggestions to improve clarity and relevance. The reliability of the instrument was also tested to ensure the consistency of the measurement. Examples of test items include contextual problems such as determining temperature changes, movement between building floors, and situations involving gains and losses represented by integers. These questions were designed to measure students' ability to apply integer concepts in real-life contexts, reflecting higher-order thinking and conceptual understanding rather than simple procedural calculations.

The Research Procedure was Conducted in Several Stages

First, a pre-test was administered to both the experimental and control classes to measure students' initial understanding of integer concepts. Second, the experimental class received instruction using visual and manipulative instructional media, such as number lines and concrete representations, while the control class received conventional instruction through explanation and textbook exercises. The learning process was conducted over several instructional sessions.

Third, after the treatment period, a post-test was administered to both groups to measure students' learning improvement. The collected data were analyzed using descriptive and inferential statistics. Descriptive statistics were used to describe students' learning outcomes in both groups, while inferential statistics were used to test the research hypothesis. An independent sample t-test was conducted to determine whether there was a significant difference in conceptual understanding between the experimental and control groups. The level of significance used in this study was 0.05.

RESULT AND DISCUSSION

The results of this study indicate that the use of instructional media has a positive and significant effect on elementary school students' understanding of integer concepts. The research subjects consisted of two groups: the experimental class, which used visual and manipulative instructional media, and the control class, which received conventional teaching methods. Based on the pre-test, the experimental class had an average score of 55.6 with a standard deviation of 7.2, while the control class had an average score of 56.3 with a standard deviation of 6.8, indicating that the initial abilities of both groups were relatively similar. After the treatment, the post-test average of the experimental class increased to 82.4 with a standard deviation of 5.9, whereas the control class increased to 68.7 with a standard deviation of 6.1. The increase in the experimental class (26.8) was higher than in the control class (12.4). An independent sample t-test revealed a significant difference between the two groups with $t = 7.21$ and $p < 0.05$.

The significant effect of visual and manipulative instructional media observed in this study is consistent with findings from recent meta-analytic research. Schoenherr et al., (2024) reported a medium overall positive effect

of visualization interventions on mathematics learning outcomes ($g = 0.504$) across a synthesis of 41 studies, with the strongest effects emerging in quasi-experimental settings and when visualizations were compared with conventional instruction. The results of the present study, which employed a quasi-experimental design with pre-test and post-test comparisons against a control group receiving conventional instruction, align closely with the conditions identified by Schoenherr et al., (2024) as most conducive to strong visualization intervention effects, reinforcing the reliability of the present findings.

The effectiveness of physical manipulative tools documented in this study also resonates with the broader evidence base on manipulative-based instruction in primary education. Byrne et al., (2023), in a comprehensive scoping review, confirmed that physical manipulative interventions consistently produce improvements in children's mathematical understanding, particularly for concepts involving abstract reasoning about numerical relationships. The present study's use of number cards, number blocks, and number lines as concrete manipulatives provided students with the hands-on, multi-sensory learning experiences that prior research identifies as critical for bridging the gap between students' informal number reasoning and the formal mathematical concepts underlying integer operations.

These results indicate that instructional media helped students visualize integer concepts, both positive and negative, making it easier to understand operations and relationships between numbers. Visual and manipulative media, such as number cards, number blocks, and number lines, provided concrete learning experiences consistent with Piaget's concrete operational stage, where elementary students understand abstract concepts more effectively through hands-on activities. Although the control class also showed improvement, it was not as significant as the experimental class, demonstrating that instructional media makes the learning process more interactive, engaging, and effective in building conceptual understanding. Therefore, visual and manipulative instructional media are proven to be effective strategies for improving students' understanding of integer concepts, especially for abstract topics such as negative numbers and integer operations.

The improvement in conceptual understanding demonstrated by the experimental class also reflects the constructivist epistemological foundation underlying manipulative-based instruction. As Greenstein & E Fernández, (2023) argued, the power of manipulative tools in mathematics education lies in their capacity to support learners' construction of abstract mathematical meaning through sensorimotor engagement with concrete materials. In the present study, students' active manipulation of number line representations and concrete counters enabled them to construct their own understanding of integer magnitude, direction, and operations through direct physical experience, rather than through passive reception of abstract symbolic rules.

The magnitude of improvement observed in the experimental class (mean gain of 26.8 points) in the present study also aligns with the findings of

Furner & NL Worrell ([2017](#)), who emphasized that hands-on learning experiences are particularly effective in fostering conceptual understanding of abstract mathematical concepts such as integers. Despite the overall positive trend in the experimental class, it is important to acknowledge that not all students demonstrated equal levels of improvement. Closer examination of individual scores suggests that certain students showed relatively limited gains even after exposure to the same instructional media. This individual variation may be attributed to differences in prior knowledge, cognitive readiness, or level of engagement during instructional sessions. Future studies should consider incorporating differentiated instruction approaches alongside visual and manipulative media to address within-class variability more systematically.

The central role of the number line in facilitating integer understanding observed in this study is supported by specific empirical evidence. [Sari et al. \(2020\)](#) found that empty number line representations are particularly helpful for supporting students' understanding of integer addition and subtraction, as they allow students to enact directional movement, thereby grounding the abstract concept of positive and negative directions in a spatial experience. Anton & Abrahamson, ([2025](#)) further demonstrated that physical engagement with number line representations supports students in developing an enactive and conceptually grounded understanding of integer arithmetic. These findings are directly consistent with the mechanism through which the number line media in the present study facilitated learning in the experimental class.

The pre-test results in the present study, which showed that both groups entered the intervention with relatively low average scores, suggest that the students had significant prior misconceptions about integer concepts. This aligns with the systematic review by Lin, ([2022](#)), who identified negative integers as among the most persistently misunderstood numerical topics at the elementary level, with students across diverse educational contexts demonstrating systematic errors related to the comparison and operation of negative numbers. The targeted use of visual and manipulative media in the experimental class appears to have been effective in addressing these prior misconceptions, as evidenced by the substantially greater post-test improvement in the experimental group compared to the control group.

The robust conceptual gains observed in the experimental class also reflect the benefits of multi-representational instruction for developing integer understanding. Lamb et al., ([2023](#)) found that students who could flexibly coordinate multiple representations of integers — including number lines, counter models, and contextual scenarios — showed deeper and more transferable conceptual understanding. In the present study, the instructional design intentionally integrated multiple representational modes, including visual number lines for understanding integer position and comparison, and concrete manipulatives for understanding integer operations, which likely contributed to the robust conceptual gains observed in the experimental class.

An important dimension of the present study's findings is the specific focus on conceptual understanding rather than procedural computation. Nurnberger-Haag et al., (2022) demonstrated that conceptual understanding and procedural accuracy are distinct dimensions of integer competence, and that students can perform integer calculations procedurally without possessing genuine understanding of why the operations work. The assessment instrument used in the present study was explicitly designed to measure conceptual understanding through contextual and interpretive tasks, ensuring that the observed improvements in the experimental class reflected genuine conceptual learning rather than superficial procedural skill acquisition.

The findings of this study also have implications for the potential integration of digital and virtual manipulative tools in integer instruction. Ismail et al., (2023) found that virtual manipulative platforms significantly improved students' performance in integer topics compared to conventional instruction, demonstrating that the benefits of manipulative-based learning can be extended to digital learning environments. Together, these findings suggest that the visual and manipulative approach demonstrated to be effective in the present study could be further enhanced through the incorporation of digital tools, a direction that merits exploration in future research.

Table 1. Pre-Test and Post-Test Results of Experimental and Control Classes

Class	Pre-Test (Average)	Standard Deviation	Post-Test (Average)	Standard Deviation	Average Increase
Experiment	55,6	7,2	82,4	5,9	26,8
Control	56,3	6,8	68,7	6,1	12,4

Source: Data Processed

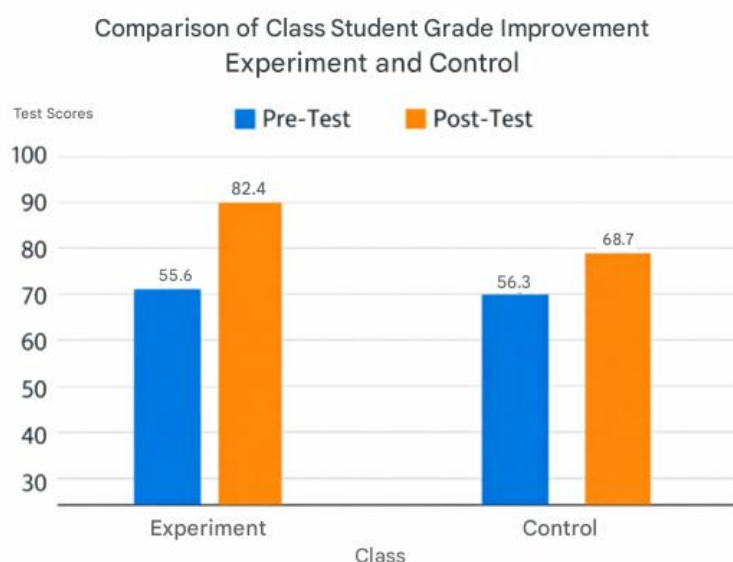


Figure 1. Comparison of Student Grade Improvement

CONCLUSION

Based on the results of this study, it can be concluded that the use of visual and manipulative instructional media has a positive and significant effect on elementary school students' understanding of integer concepts, with the experimental class showing greater improvement than the control class. Number line representations are most effective for teaching integer ordering and comparison, while manipulatives such as two-colored counters and balance models are most effective for integer operations involving negative numbers. Teachers are encouraged to implement these tools sequentially — from conceptual orientation through number lines, to operational understanding through manipulatives, and finally to contextual problem-solving tasks. Schools should support this approach through adequate resources and professional development, and future research is recommended to explore the effectiveness of digital learning tools in supporting students' conceptual understanding of integers.

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